

Test of the weak cosmic censorship conjecture with a charged scalar field and dyonic Kerr-Newman black holes

Gábor Zsolt Tóth*

Institute for Particle and Nuclear Physics
Wigner Research Centre for Physics, Hungarian Academy of Sciences
Konkoly Thege Miklós út 29-33
H-1121 Budapest, Hungary

Abstract

A thought experiment considered recently in the literature, in which it is investigated whether a dyonic Kerr-Newman black hole can be destroyed by overcharging or overspinning it past extremality by a massive complex scalar test field, is revisited. Another derivation of the result that this is not possible, i.e. the weak cosmic censorship is not violated in this thought experiment, is given. The derivation is based on conservation laws, on a null energy condition, and on specific properties of the metric and the electromagnetic field of dyonic Kerr-Newman black holes. The metric is kept fixed, whereas the dynamics of the electromagnetic field is taken into account. A detailed knowledge of the solutions of the equations of motion is not needed. The approximation in which the electromagnetic field is fixed is also considered, and a derivation for this case is also given. In addition, an older version of the thought experiment, in which a pointlike test particle is used, is revisited. The same result, namely the non-violation of the cosmic censorship, is rederived in a way which is simpler than in earlier works.

*email: tgzs@rmki.kfki.hu

1 Introduction

If a spacetime contains a singularity not hidden behind an event horizon (a “naked” singularity), then far away observers can receive signals coming from this singularity. However, initial conditions cannot be specified at a singularity, therefore a singularity that is not behind an event horizon prevents predictability in the spacetime that contains it. For this reason, it is conjectured that naked singularities cannot be produced in a physical process from regular initial conditions, if the matter involved in the process has reasonable properties. This conjecture, first stated by Penrose [1], is known as the weak cosmic censorship conjecture (WCCC) (for a textbook exposition, see e.g. section 12.1 of [2]), and it is one of the major unsolved problems of classical general relativity to decide whether it is correct.

In the absence of a general proof, the validity of the WCCC has been checked in several special cases by studying the evolution of initially regular physical systems. A possible test is to throw a small particle at a Kerr-Newman black hole and to see if an overextremal Kerr-Newman spacetime, which contains a naked singularity, can arise after the particle has been absorbed by the black hole. Wald [3] was the first who considered this thought experiment, and he showed that an extremal Kerr-Newman black hole cannot be overcharged or overspin by throwing a pointlike test particle with electric charge into it. A simpler derivation of this result was given by Needham [4]. Hiscock [5] and Semiz [6] extended Wald’s result to the case of dyonic Kerr-Newman black holes, which are rotating black holes with both electric and magnetic charge. The derivations presented in [5, 6] are generalizations of the derivation in [3]. Recently Semiz [7] also studied the case when a complex scalar test field is used instead of a test particle, and found that the WCCC is not violated. Other results supporting the WCCC were obtained by several authors, and there are also claims that the WCCC can be violated, for example by starting from a slightly sub-extremal black hole and “jumping over” the extremal case [17]-[37]. The signatures of naked singularities for the observational verification of their existence was also investigated, e.g. in [38, 39]. Reviews on the status of the cosmic censorship conjecture can be found in [40]-[44].

A dyonic Kerr-Newman black hole can be characterized by four parameters, which are the mass M , the angular momentum per unit mass a , the electric charge Q_e and the magnetic charge Q_m . The angular momentum of the black hole is $J = aM$, and $Q_m = 0$ corresponds to a usual Kerr-Newman black hole. The metric of the dyonic Kerr-Newman black hole spacetime with parameters (M, a, Q_e, Q_m) is the same as the Kerr-Newman metric with parameters (M, a, e) , $e^2 = Q_e^2 + Q_m^2$, where e denotes the electric charge parameter of the Kerr-Newman metric. The parameters have to satisfy the inequality

$$\eta = M^2 - Q_e^2 - Q_m^2 - a^2 \geq 0, \quad (1)$$

otherwise the spacetime contains a naked singularity. The black hole is called extremal if $\eta = 0$. Under certain conditions, the dyonic Kerr-Newman black holes are the only static and asymptotically flat black hole solutions of the Einstein-Maxwell equations [9, 10].

Under a small change (dM, dJ, dQ_e, dQ_m) of the parameters of the black hole, the change of η is

$$d\eta = 2 \frac{M^2 + a^2}{M} \left(dM - \frac{a}{M^2 + a^2} dJ - \frac{Q_e M}{M^2 + a^2} dQ_e - \frac{Q_m M}{M^2 + a^2} dQ_m \right). \quad (2)$$

In the thought experiments discussed in [3, 4, 5, 6, 7] it is assumed that initially one has an extremal dyonic Kerr-Newman black hole, which then absorbs a small amount of matter, and finally settles down in another dyonic Kerr-Newman state with slightly different parameters. If one calculates the change (dM, dJ, dQ_e, dQ_m) of the parameters in this process, one should find $d\eta \geq 0$; a result $d\eta < 0$ would indicate a possible violation of the WCCC. In [3, 4, 5, 6, 7] the change (dM, dJ, dQ_e, dQ_m) of the parameters were calculated in the approximation that the metric is fixed during the process and, as mentioned above, the result $d\eta \geq 0$ was found.

In [3, 4, 5, 6], where the thought experiment with a pointlike test particle is considered, not only the metric but also the electromagnetic field is taken to be fixed in the calculation of (dM, dJ, dQ_e, dQ_m) . In [7], however, where the case of the test field is considered, the electromagnetic field is also taken to be dynamical, although with the restriction that free electromagnetic radiation that is not tied to the electric current is not present.

In this paper we revisit the thought experiment studied by Semiz [7], in which it is investigated whether a dyonic Kerr-Newman black hole can be destroyed by overcharging or overspinning it past extremality by a massive complex scalar test field. We give a different and simpler derivation of the result that $d\eta \geq 0$ in the extremal case, which indicates that the black hole remains intact and thus the WCCC is not violated. Our derivation is based on conservation laws, on a null energy condition, and on certain specific properties of the metric and the electromagnetic field of dyonic Kerr-Newman black holes. In contrast with the derivation in [7], in our derivation it is not necessary to have a detailed knowledge of the solutions of the equations of motion, and we do not impose the restriction on the electromagnetic field that is imposed in [7] either.

We also consider the older version of the thought experiment in which a pointlike test particle is applied, and we present a slightly different and simpler derivation than those in [5, 6] for this case as well. This is an extension of Needham's derivation [4] for Kerr-Newman black holes to the dyonic case.

In these derivations we make the same approximations as are made in [3, 4, 5, 6, 7], i.e. in the case of the pointlike test particle both the metric and the electromagnetic

field is kept fixed, whereas in the case of the test field only the metric is kept fixed. Nevertheless, one can also consider the scalar test field in fixed gravitational and electromagnetic fields, therefore we present a derivation for this case as well. Apart from certain differences, this derivation is similar to the one for the case of non-fixed electromagnetic field.

In our arguments we do not restrict ourselves to extremal black holes, but rather we derive an inequality in each case that is valid for any values of the black hole parameters and that gives the desired result in the extremal case.

It is worth noting that processes similar to those mentioned above are dealt with in the derivation of “physical process versions” of the first law of black hole mechanics [11]-[16]. These derivations are similar to some extent to those that we present for the case of the scalar field, nevertheless the dynamics of the metric has an important role in them.

The paper is organized as follows. In section 2 further necessary information about the dyonic Kerr-Newman black holes is recalled. In section 3 the thought experiment with pointlike particle is discussed and the derivation of the result that the cosmic censorship is not violated is given. In section 4 the version of the thought experiment in which a scalar field is used is considered, and our derivations of the non-violation of the cosmic censorship for this case are presented. In the Appendix Noether’s theorem, as it is used in section 4, is described.

2 The dyonic Kerr-Newman black holes

The metric of the dyonic Kerr-Newman black hole spacetime with parameters (M, a, Q_e, Q_m) can be given in a standard form as

$$\begin{aligned}
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = & \\
& - \left(\frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) dt^2 - \frac{2a \sin^2 \theta (r^2 + a^2 - \Delta)}{\Sigma} dt d\phi \\
& + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(\frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2,
\end{aligned} \tag{3}$$

where

$$\Sigma = r^2 + a^2 \cos^2 \theta \tag{4}$$

$$\Delta = r^2 + a^2 + Q_e^2 + Q_m^2 - 2Mr. \tag{5}$$

The signature of this metric is $(-+++)$.

The electromagnetic field of a dyonic Kerr-Newman black hole has the vector potential

$$A = Q_e A_e + Q_m A_m, \tag{6}$$

where

$$A_e = -\frac{r}{\Sigma}dt + \frac{ar \sin^2 \theta}{\Sigma}d\phi \quad (7)$$

and

$$A_m = \frac{a \cos \theta}{\Sigma}dt + \left[\tilde{C} - \cos \theta \frac{r^2 + a^2}{\Sigma} \right] d\phi. \quad (8)$$

The electromagnetic field derived from A_m is dual to the electromagnetic field derived from A_e . The electromagnetic field does not depend on the constant \tilde{C} , which can be used, by setting $\tilde{C} = 1$ or $\tilde{C} = -1$, to eliminate the Dirac string singularity of A_m along the positive or negative z axis ($\theta = 0$ and $\theta = \pi$), respectively. We set \tilde{C} to zero for a reason that is explained below.

In the following sections various quantities will be considered at the future event horizon. Since the Boyer-Lindquist coordinates (t, r, θ, ϕ) do not cover the future event horizon, Eddington-Finkelstein-type ingoing horizon-penetrating coordinates, denoted by $(\tau, r, \theta, \varphi)$, will be used. These coordinates can be introduced by the transformation

$$\tau = t - r + \int dr \frac{r^2 + a^2}{\Delta} \quad (9)$$

$$\varphi = \phi + \int dr \frac{a}{\Delta}. \quad (10)$$

The future event horizon is located in these coordinates at the constant value

$$r_+ = M + \sqrt{M^2 - (a^2 + Q_e^2 + Q_m^2)} \quad (11)$$

of r , and the metric is non-singular in these points. In the extremal case

$$r_+ = M. \quad (12)$$

The $(\tau + r, \theta, \varphi) = \text{constant}$ lines are ingoing null geodesics, and there exists an $r_0 < r_+$ such that the $\tau = \text{constant}$ hypersurfaces are spacelike in the domain $r_0 < r$.

The r component $(A_e)_r$ of A_e with respect to the coordinates $(\tau, r, \theta, \varphi)$ is singular at the event horizon, but this singularity can be eliminated by the gauge transformation $A_e \rightarrow A_e - \frac{r}{\Delta}dr$. After this gauge transformation

$$A_e = -\frac{r}{\Sigma}d\tau + \frac{ar \sin^2 \theta}{\Sigma}d\varphi - \frac{r}{\Sigma}dr. \quad (13)$$

The r component of A_m with respect to the coordinates $(\tau, r, \theta, \varphi)$ is also singular if $\tilde{C} \neq 0$, therefore we set $\tilde{C} = 0$. Nevertheless, in order to treat the Dirac string

singularity of A_m , we introduce an explicit gauge parameter into it by adding $Cd\varphi$, where C is a real constant. Thus

$$A_m = \frac{a \cos \theta}{\Sigma} d\tau + \left[C - \cos \theta \frac{r^2 + a^2}{\Sigma} \right] d\varphi + \frac{a \cos \theta}{\Sigma} dr. \quad (14)$$

A_m has a string singularity along the z axis (which corresponds to $\theta = 0$ and $\theta = \pi$) because $d\varphi$ is singular here, and its coefficient $(A_m)_\varphi$ does not cancel this singularity. However, in the special cases $C = 1$ and $C = -1$ the singularity is cancelled along the positive z axis ($\theta = 0$) or along the negative z axis ($\theta = \pi$), respectively. The string singularity can therefore be avoided by using two domains that cover the whole spacetime region of interest in such a way that one of the domains contains the entire positive z axis but is well separated from the negative z axis and the other one contains the entire negative z axis but is separated from the positive z axis. In the first domain the $C = 1$ gauge is used then, and in the second domain the $C = -1$ gauge. Suitable domains are given by $r_0 < r$, $0 \leq \theta \leq \pi/2$ and $r_0 < r$, $\pi/2 < \theta \leq \pi$, for example. These domains will be denoted by \mathcal{D}_+ and \mathcal{D}_- . The transition between the two domains involves a gauge transformation, which has to be kept in mind in particular calculations. This approach to treating the string singularity of A_m was proposed in [8] and was also taken in [6, 7].

In the rest of the paper we use only the coordinates $(\tau, r, \theta, \varphi)$, and we also use the notation ω for the one-form dr (the exterior derivative of the coordinate function r). A_e , A_m and A will denote (13), (14) and $A = Q_e A_e + Q_m A_m$, respectively.

$\partial/\partial\tau$ and $\partial/\partial\varphi$ are Killing fields; $\partial/\partial\tau$ is the generator of time translations and $\partial/\partial\varphi$ is the generator of rotations around the axis of the black hole. A_e and A_m are also invariant under these symmetries. The Killing field

$$\chi = \frac{\partial}{\partial\tau} + \Omega_H \frac{\partial}{\partial\varphi} \quad (15)$$

is null at the event horizon, with

$$\Omega_H = \frac{a}{r_+^2 + a^2}, \quad (16)$$

which is called the angular velocity of the event horizon. At the event horizon we also have

$$(A_e)_\mu \chi^\mu = \frac{-r_+}{r_+^2 + a^2}, \quad (A_m)_\mu \chi^\mu = C\Omega_H, \quad (17)$$

and ω^μ is parallel to χ^μ . The relation between ω^μ and χ^μ at the event horizon is

$$\omega^\mu = \frac{r_+^2 + a^2}{r_+^2 + a^2 \cos^2 \theta} \chi^\mu, \quad (18)$$

thus ω^μ is also future directed. We introduce the quantity Φ_H as

$$\Phi_H = \frac{r_+ Q_e}{r_+^2 + a^2} . \quad (19)$$

In the case of Kerr-Newman black holes, Φ_H is known as the electrostatic potential of the horizon.

Both \mathcal{D}_+ and \mathcal{D}_- are contractible domains, therefore any gauge transformation takes the form $A \rightarrow A + d\Phi$ on \mathcal{D}_+ or on \mathcal{D}_- , where Φ denotes a real valued function. If $d\Phi$ is invariant under the time translation and rotation symmetries of the spacetime, then Φ takes the form $\Phi_0(r, \theta) + c_1\tau + c_2\varphi$, where c_1 and c_2 are constants, thus the corresponding gauge transformation changes the τ and φ components of A only by adding the constants c_1 and c_2 . This shows that if A (or A_e, A_m), understood here to be fixed up to gauge transformations, is required to be invariant under time translation and rotation, then the τ and φ components of A are determined uniquely up to additive constants. These constants can also be fixed by requiring that A_τ should tend to 0 as $r \rightarrow \infty$ and A should not have a Dirac string singularity. If the τ and φ components of A are fixed, then the quantity $A_\mu \chi^\mu$ is also fixed, because it depends only on these components of A .

3 Thought experiment with a point particle

In this section we consider the thought experiment in which a pointlike test particle is thrown at a dyonic Kerr-Newman black hole. The extremality of the black hole is not assumed; we derive an inequality that holds for any values of the black hole parameters, and that becomes the desired inequality $d\eta > 0$ in the extremal case. The argument that we present is an extension of the argument of [4] for Kerr-Newman black holes to the more general case of dyonic Kerr-Newman black holes.

As explained in [6], the magnetic charge of the test particle can always be set to zero by a duality rotation, therefore it can be assumed without loss of generality that the test particle has zero magnetic charge.

The Lagrangian of the test particle with mass m , electric charge q and zero magnetic charge is

$$\mathcal{L} = \frac{1}{2} m g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} + q A_\mu \frac{dx^\mu}{ds} , \quad (20)$$

and its conserved energy and angular momentum are

$$E = -m g_{\tau\mu} \frac{dx^\mu}{ds} - q A_\tau \quad (21)$$

$$L = +m g_{\varphi\mu} \frac{dx^\mu}{ds} + q (A_\varphi - Q_m C) . \quad (22)$$

The $-qQ_mC$ term on the right hand side is added to cancel the dependence of A_φ on the gauge parameter C . This is important because C has different values in the two domains \mathcal{D}_+ and \mathcal{D}_- .

By multiplying (22) by Ω_H and then subtracting it from (21), and taking into account (15), (16), (17) and (19), it follows immediately that if the particle does cross the event horizon, then

$$-m\frac{dx^\mu}{ds}\chi_\mu = E - \Omega_H L - \Phi_H q \quad (23)$$

holds at the point where the crossing takes place. At this point $\frac{dx^\mu}{ds}\chi_\mu < 0$ also obviously holds, since $\frac{dx^\mu}{ds}$ is a timelike future directed vector in the case of a massive particle, and χ^μ is a future directed null vector at the event horizon. Thus,

$$E - \Omega_H L - \Phi_H q > 0 . \quad (24)$$

We note that $\frac{dx^\mu}{ds}\chi_\mu$ is just the r component of $\frac{dx^\mu}{ds}$ multiplied by a positive number, as can be seen from (18), and the r component of $\frac{dx^\mu}{ds}$ is clearly negative or zero for a particle moving inward into the black hole.

The change of the black hole parameters dM , dJ and dQ_e in (2) can be identified with E , L and q , respectively, and $dQ_m = 0$. The inequality (24) together with the relations $dM = E$, $dJ = L$, $dQ_e = q$ and $dQ_m = 0$ imply

$$dM - \Omega_H dJ - \Phi_H dQ_e > 0 . \quad (25)$$

In the extremal case $\Omega_H = \frac{a}{M^2+a^2}$ and $\Phi_H = \frac{MQ_e}{M^2+a^2}$, thus in this case (2) can be written as $dM - \Omega_H dJ - \Phi_H dQ_e = \frac{M}{2(M^2+a^2)}d\eta$, therefore in the extremal case (25) gives $d\eta > 0$, which indicates that the black hole is not destroyed by the absorption of the test particle, i.e. no violation of the WCCC occurs.

4 Thought experiment with test fields

In this section we consider a similar thought experiment as in section 3, but with test fields instead of a pointlike test particle. We discuss two different settings, in sections 4.1 and 4.2, respectively.

In section 4.1 we consider the process in which a small amount of electrically charged matter, described by a complex scalar field ψ , falls into a dyonic Kerr-Newman black hole. As in section 3, we make the approximation in the calculation of dM , dJ and dQ_e that the metric and the electromagnetic field do not change, i.e. we take the scalar test field to evolve in the fixed gravitational and electromagnetic background fields of the black hole.

In section 4.2 a similar process is considered, with the difference that only the gravitational field is kept fixed, which means that the test matter also has an electromagnetic field component and the effect of the scalar field on the electromagnetic field is taken into account. This is the setting that was considered in [7], although in [7] the difference of the total electromagnetic field and the electromagnetic field of the black hole is tied to the electric current (see [7] for details). We do not impose such a restriction on the electromagnetic field.

Our reason for considering also the first case, in which the electromagnetic field is fixed, is that this is the one that is obtained from the thought experiment described in section 3 if the pointlike test particle is replaced by a scalar test field in a straightforward manner, and it also has technical differences from the second case.

One of the technical differences between the two settings is that the Einstein-Hilbert energy-momentum tensor is conserved and can be used to obtain the conserved energy and angular momentum currents only in the second setting. For this reason in the first setting we use Noether's theorem to find the conserved currents. Noether's theorem can be used in the second setting as well; the currents obtained in this way differ only in divergence terms from the currents obtained from the Einstein-Hilbert energy-momentum tensor, and these terms do not give any contribution to dM and dJ .

In the same way as in section 3, the extremality of the black hole is generally not assumed in sections 4.1 and 4.2.

4.1 Scalar test field

The action of the scalar field in fixed dyonic Kerr-Newman gravitational and electromagnetic fields is $\mathcal{S} = \int \sqrt{-g} \mathcal{L} d\tau dr d\theta d\varphi$, with the Lagrangian density

$$\mathcal{L} = -g^{\mu\nu}(\partial_\mu - ieA_\mu)\psi^*(\partial_\nu + ieA_\nu)\psi - m^2\psi^*\psi, \quad (26)$$

where A_μ is the vector potential of the electromagnetic field of the black hole as given in section 2.

The energy and angular momentum Noether currents corresponding to the symmetries generated by $\partial/\partial\tau$ and $\partial/\partial\varphi$ are

$$\sqrt{-g}\mathcal{E}^\mu = \sqrt{-g}T^\mu{}_\nu(\partial/\partial\tau)^\nu = \sqrt{-g}T^\mu{}_\tau \quad (27)$$

and

$$\sqrt{-g}\mathcal{J}^\mu = \sqrt{-g}T^\mu{}_\nu(\partial/\partial\varphi)^\nu = \sqrt{-g}T^\mu{}_\varphi, \quad (28)$$

where $T^\mu{}_\nu$ is

$$\begin{aligned} T^\mu{}_\nu &= -\frac{\partial\mathcal{L}}{\partial_\mu\psi}\partial_\nu\psi - \frac{\partial\mathcal{L}}{\partial_\mu\psi^*}\partial_\nu\psi^* + \delta^\mu{}_\nu\mathcal{L} \\ &= (\partial^\mu - ieA^\mu)\psi^*\partial_\nu\psi + (\partial^\mu + ieA^\mu)\psi\partial_\nu\psi^* + \delta^\mu{}_\nu\mathcal{L} \end{aligned} \quad (29)$$

(see Appendix A for more details on Noether's theorem). The conservation laws for these currents are $\partial_\mu(\sqrt{-g}\mathcal{E}^\mu) = 0$ and $\partial_\mu(\sqrt{-g}\mathcal{J}^\mu) = 0$. $T^\mu{}_\nu$ is introduced only for notational convenience.

The Noether current corresponding to the $\psi \rightarrow e^{i\alpha}\psi$, $\alpha \in \mathbb{R}$ global $U(1)$ symmetry is $\sqrt{-g}j^\mu$, where

$$j^\mu = ie[\psi^*(\partial^\mu + ieA^\mu)\psi - \psi(\partial^\mu - ieA^\mu)\psi^*] \quad (30)$$

is the electric current. j^μ also satisfies the equality $j^\mu = \frac{\partial \mathcal{L}}{\partial A_\mu}$.

$T^\mu{}_\nu$, \mathcal{E}^μ , \mathcal{J}^μ and j^μ are quantities that transform as proper tensor and vector fields, respectively, under coordinate transformations. The conservation laws can be written, of course, as $\nabla_\mu \mathcal{E}^\mu = 0$, $\nabla_\mu \mathcal{J}^\mu = 0$ and $\nabla_\mu j^\mu = 0$.

Defining $\hat{T}_{\mu\nu}$ as

$$\hat{T}_{\mu\nu} = (\partial_\mu - ieA_\mu)\psi^*(\partial_\nu + ieA_\nu)\psi + (\partial_\mu + ieA_\mu)\psi(\partial_\nu - ieA_\nu)\psi^* + g_{\mu\nu}\mathcal{L}, \quad (31)$$

we have

$$T_{\mu\nu} = \hat{T}_{\mu\nu} + A_\nu j_\mu, \quad (32)$$

and $\mathcal{E}^\mu = \hat{T}^\mu{}_\tau + A_\tau j^\mu$, $\mathcal{J}^\mu = \hat{T}^\mu{}_\varphi + A_\varphi j^\mu$. $\hat{T}_{\mu\nu}$ and j^μ are gauge invariant and A_τ does not depend on the gauge parameter C , therefore \mathcal{E}^μ is also independent of C . A_φ does depend on C , however, thus \mathcal{J}^μ also depends on it. For this reason we take the modified definition

$$\mathcal{J}^\mu = \hat{T}^\mu{}_\varphi + (A_\varphi - Q_m C)j^\mu \quad (33)$$

for \mathcal{J}^μ , which eliminates its dependence on C . The conservation of \mathcal{J}^μ is not affected by this modification, because j^μ is conserved. The independence of \mathcal{E}^μ and \mathcal{J}^μ of C is important because the value of C is different in the domains \mathcal{D}_+ and \mathcal{D}_- .

The charge flux through the event horizon into the black hole is

$$\frac{dQ}{d\tau} = - \int_H \sqrt{-g} j^r d\theta d\varphi, \quad (34)$$

where H denotes the two-dimensional surface of the black hole (which is the relevant time slice of the event horizon), and the energy and angular momentum fluxes are

$$\frac{dE}{d\tau} = \int_H \sqrt{-g} [\hat{T}^r{}_\tau + A_\tau j^r] d\theta d\varphi \quad (35)$$

$$\frac{dL}{d\tau} = - \int_H \sqrt{-g} [\hat{T}^r{}_\varphi + (A_\varphi - Q_m C)j^r] d\theta d\varphi, \quad (36)$$

where the quantities in the brackets are \mathcal{E}^r and \mathcal{J}^r , respectively. The total energy, angular momentum and charge that falls through the event horizon is $\int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau$, $\int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau$ and $\int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$, respectively.

One of the main assumptions of the thought experiment is that the final state of the physical system is again a dyonic Kerr-Newman state, which means that all matter that does not fall through the event horizon is assumed to escape eventually to infinity. In particular, it is assumed that the energy, angular momentum and charge of the matter contained in the domain $r_+ \leq r \leq r_m$, given by the integrals $-\int_{r_+}^{r_m} dr \int \sqrt{-g} \mathcal{E}^\tau d\theta d\varphi$, $\int_{r_+}^{r_m} dr \int \sqrt{-g} \mathcal{J}^\tau d\theta d\varphi$, $\int_{r_+}^{r_m} dr \int \sqrt{-g} j^\tau d\theta d\varphi$, go to 0 as $\tau \rightarrow \infty$ for any fixed value of r_m . Under this assumption dM , dJ and dQ_e can be identified with $\int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau$, $\int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau$ and $\int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$, i.e. the change of the mass, angular momentum and electric charge of the black hole equals to the total energy, angular momentum and electric charge that falls through the event horizon.

From the equations (34), (35), (36) above and from (15), (16), (17) and (19) it follows immediately that

$$\int_H \sqrt{-g} \hat{T}_{\mu\nu} \omega^\mu \chi^\nu d\theta d\varphi = \frac{dE}{d\tau} - \Omega_H \frac{dL}{d\tau} - \Phi_H \frac{dQ}{d\tau} . \quad (37)$$

Taking into account the relations $dM = \int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau$, $dJ = \int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau$ and $dQ_e = \int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$,

$$\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \omega^\mu \chi^\nu d\theta d\varphi = dM - \Omega_H dJ - \Phi_H dQ_e \quad (38)$$

is obtained from (37). The right hand side in (38) is the same as the left hand side in (25), and in the extremal case it is $\frac{M}{2(M^2+a^2)} d\eta$. Thus the sign of $d\eta$ depends, in the extremal case, on the sign of $\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \omega^\mu \chi^\nu d\theta d\varphi$.

From the fact that χ^μ is a null vector at the event horizon and from the form (31) of $\hat{T}_{\mu\nu}$ it is obvious that at the event horizon $\hat{T}_{\mu\nu}$ satisfies the inequality $\hat{T}_{\mu\nu} \chi^\mu \chi^\nu \geq 0$ for arbitrary A_μ and ψ functions. Taking into consideration (18), this also means that $\hat{T}_{\mu\nu} \omega^\mu \chi^\nu \geq 0$ holds for the integrand $\hat{T}_{\mu\nu} \omega^\mu \chi^\nu$ on the left hand side of (38), hence

$$dM - \Omega_H dJ - \Phi_H dQ_e \geq 0 . \quad (39)$$

In particular, in the extremal case $d\eta \geq 0$, indicating that the WCCC is not violated. The inequality (39) is almost identical to (25), the only minor difference is that in (39) equality is allowed.

Regarding the condition of strict equality in (39), $dM - \Omega_H dJ - \Phi_H dQ_e = 0$ holds if and only if $\chi^\mu (\partial_\mu + ieA_\mu) \psi = 0$ everywhere on the future part of the event horizon. It is easy to see that in this case the charge, energy and angular momentum fluxes $\frac{dQ}{d\tau}$, $\frac{dE}{d\tau}$ and $\frac{dL}{d\tau}$ into the black hole are zero, thus $dM = dJ = dQ_e = 0$. Furthermore, since $\chi^\mu A_\mu$ is a real constant on the event horizon, the equation $\chi^\mu (\partial_\mu + ieA_\mu) \psi = 0$ implies that either $\psi = 0$ everywhere on the future part of the event horizon, or ψ is

of the form $\psi = \psi_0 e^{i\alpha\tau}$, where $\alpha \in \mathbb{R}$ and $\psi_0 \neq 0$, along some integral curves of χ^μ . The second possibility is excluded if we assume $\lim_{\tau \rightarrow -\infty} \psi = 0$ at the event horizon.

We close this section with two remarks. Although the expressions $\mathcal{E}^\mu = \hat{T}^\mu{}_\tau + A_\tau j^\mu$ and $\mathcal{J}^\mu = \hat{T}^\mu{}_\varphi + (A_\varphi - Q_m C)j^\mu$ appear to be gauge dependent because of the explicit presence of A_τ and A_φ in them, it is important to note that the vector potential A that is used in these expressions is invariant under time translations and rotations, which is a property that is also used when Noether's theorem is applied, and which fixes A_τ and A_φ uniquely (up to additive constants), as mentioned in section 2. Furthermore, if A is replaced by some gauge transformed vector potential $A + d\Phi$ in the Lagrangian (26), then the quantity K^μ appearing in the invariance condition (A.3) also has to be modified as $K^\mu \rightarrow K^\mu + j^\mu \partial_\tau \Phi$ or $K^\mu \rightarrow K^\mu + j^\mu \partial_\varphi \Phi$ (for time translations and rotations, respectively), where j^μ is the electric current. The effect of this modification is that the A_τ and A_φ appearing explicitly in the formulas $\mathcal{E}^\mu = \hat{T}^\mu{}_\tau + A_\tau j^\mu$ and $\mathcal{J}^\mu = \hat{T}^\mu{}_\varphi + (A_\varphi - Q_m C)j^\mu$ remain unchanged. Of course, the vector potential in the expressions for $\hat{T}^\mu{}_\tau$ and j^μ will be the gauge transformed one. The expressions (21) and (22) for the conserved energy and angular momentum in the case of the pointlike test particle can also be derived using Noether's theorem, and an argument analogous to the one above shows that the A_τ and A_φ appearing in these expressions are also well defined.

The tensor $\hat{T}_{\mu\nu}$ coincides with the Einstein-Hilbert energy-momentum tensor $-2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}$ obtained from the Lagrangian (26), and the inequality $\hat{T}_{\mu\nu}\chi^\mu\chi^\nu \geq 0$ used above has the form of a null energy condition. One might think that the energy and angular momentum currents should be defined as $\mathcal{E}^\mu = \hat{T}^\mu{}_\tau$ and $\mathcal{J}^\mu = \hat{T}^\mu{}_\varphi$, however, these currents are not conserved, which can be seen by considering that their conservation would imply the conservation of $A_\tau j^\mu$ and $A_\varphi j^\mu$. The non-conservation of these currents also means that $\nabla_\mu \hat{T}^\mu{}_\nu \neq 0$. By looking at the derivation of the conservation of the Einstein-Hilbert energy-momentum tensor (see e.g. section E.1 of [2] around equation (E.1.27)) it can be seen that the obstacle to the conservation of $\hat{T}_{\mu\nu}$ is the presence of the fixed electromagnetic field.

4.2 Scalar and electromagnetic test fields

The Lagrangian density of the scalar and electromagnetic fields in Kerr-Newman spacetime is

$$\mathcal{L} = -g^{\mu\nu}(\partial_\mu - ie\tilde{A}_\mu)\psi^*(\partial_\nu + ie\tilde{A}_\nu)\psi - m^2\psi^*\psi - \frac{1}{16\pi}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (40)$$

where $\tilde{F}_{\mu\nu} = \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu$ and the tilde is used to distinguish the vector potential of the full electromagnetic field from the vector potential of the electromagnetic field of

the dyonic Kerr-Newman black hole introduced in section 2. The electric current is

$$j^\mu = ie[\psi^*(\partial^\mu + ie\tilde{A}^\mu)\psi - \psi(\partial^\mu - ie\tilde{A}^\mu)\psi^*]. \quad (41)$$

In the present setting the Einstein-Hilbert energy-momentum tensor

$$\begin{aligned} T_{\mu\nu} &= -2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L} \\ &= (\partial_\mu - ie\tilde{A}_\mu)\psi^*(\partial_\nu + ie\tilde{A}_\nu)\psi + (\partial_\mu + ie\tilde{A}_\mu)\psi(\partial_\nu - ie\tilde{A}_\nu)\psi^* \\ &\quad + \frac{1}{4\pi}\tilde{F}_{\mu\lambda}\tilde{F}_\nu{}^\lambda + g_{\mu\nu}\mathcal{L} \end{aligned} \quad (42)$$

is conserved (i.e. $\nabla_\mu T^\mu{}_\nu = 0$) and is suitable for defining the energy and angular momentum currents as

$$\mathcal{E}^\mu = T^\mu{}_\tau, \quad \mathcal{J}^\mu = T^\mu{}_\varphi. \quad (43)$$

These currents are conserved (i.e. $\nabla_\mu \mathcal{E}^\mu = 0$ and $\nabla_\mu \mathcal{J}^\mu = 0$) because $\partial/\partial\tau$ and $\partial/\partial\varphi$ are Killing vectors and $\nabla_\mu T^\mu{}_\nu = 0$. \mathcal{E}^μ and \mathcal{J}^μ are also clearly gauge invariant. The same definition is taken for the energy and angular momentum currents in [7].

The charge, energy and angular momentum fluxes through the event horizon are given by

$$\frac{dQ}{d\tau} = - \int_H \sqrt{-g} j^r d\theta d\varphi \quad (44)$$

$$\frac{dE}{d\tau} = \int_H \sqrt{-g} T^r{}_\tau d\theta d\varphi \quad (45)$$

$$\frac{dL}{d\tau} = - \int_H \sqrt{-g} T^r{}_\varphi d\theta d\varphi. \quad (46)$$

We assume that the field ψ goes to zero as $\tau \rightarrow \infty$, in accordance with the fundamental assumption that the final state of the physical process under consideration is a dyonic Kerr-Newman state. The vector potential, on the other hand, will become $A + dQ_e A_e$ (up to gauge transformation) as $\tau \rightarrow \infty$ due to the change dQ_e of the charge of the black hole. This change of the electromagnetic field implies that the energy and the angular momentum of the electromagnetic field around the black hole also changes, which has to be taken into account in the calculation of dM and dJ . Thus dM and dJ are given by

$$\begin{aligned} dM &= \int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau - \int_{r_+}^{\infty} dr \int \sqrt{-g} T^\tau{}_\tau|_{\tilde{A}=A+dQ_e A_e, \psi=0} d\theta d\varphi \\ &\quad + \int_{r_+}^{\infty} dr \int \sqrt{-g} T^\tau{}_\tau|_{\tilde{A}=A, \psi=0} d\theta d\varphi \end{aligned} \quad (47)$$

and

$$\begin{aligned}
dJ = & \int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau + \int_{r_+}^{\infty} dr \int \sqrt{-g} T^\tau{}_\varphi|_{\tilde{A}=A+dQ_e A_e, \psi=0} d\theta d\varphi \\
& - \int_{r_+}^{\infty} dr \int \sqrt{-g} T^\tau{}_\varphi|_{\tilde{A}=A, \psi=0} d\theta d\varphi .
\end{aligned} \tag{48}$$

The second term on the right hand side of (47) and (48) gives the energy and angular momentum, respectively, of the electromagnetic field around the black hole at $\tau \rightarrow \infty$, whereas the third terms give the energy and angular momentum of the electromagnetic field around the black hole in the initial state. dQ_e is given by $dQ_e = \int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$, as in section 4.1. We note that in [7] the change of the energy and angular momentum of the electromagnetic field around the black hole is included in dM and dJ by considering fluxes through spherical surfaces of radius $r \rightarrow \infty$ rather than $r = r_+$.

Aiming to derive an equation similar to (38), we consider now the quantity $dM - \Omega_H dJ$. From (45), (46) and (15) it is easy to see that the contribution of the first terms on the right hand side of (47) and (48) to $dM - \Omega_H dJ$ is $\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} T_{\mu\nu} \omega^\mu \chi^\nu d\theta d\varphi$. Since A_e and A_m are known explicitly, the contribution of the second and third terms on the right hand side of (47) and (48) can also be evaluated. This task can be simplified by partial integrations and by using the properties of A_e , A_m and of the corresponding electromagnetic fields. In addition, those terms that are higher than first order in dQ_e should be neglected. One finds that all integrals can be evaluated trivially except for one integral over θ , and the final result is that the contribution of the terms in question is $\Phi_H dQ_e$. Thus

$$\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} T_{\mu\nu} \omega^\mu \chi^\nu d\theta d\varphi = dM - \Omega_H dJ - \Phi_H dQ_e , \tag{49}$$

which is analogous to (38) in section 4.1. From the fact that χ^μ is a null vector at the event horizon and from the form (42) of $T_{\mu\nu}$ it is obvious that at the event horizon $T_{\mu\nu}$ satisfies the inequality $T_{\mu\nu} \chi^\mu \chi^\nu \geq 0$ —a null energy condition—for arbitrary A_μ and ψ functions. Taking into consideration (18), this implies that $T_{\mu\nu} \omega^\mu \chi^\nu \geq 0$ also holds for the integrand on the left hand side of (49), hence

$$dM - \Omega_H dJ - \Phi_H dQ_e \geq 0. \tag{50}$$

This inequality, which has the same form as (39), implies $d\eta \geq 0$ in the case when the black hole is extremal, indicating that the cosmic censorship is not violated.

We note that Noether's theorem gives the conserved energy and angular momentum currents

$$\mathcal{E}^\mu = T^\mu{}_\tau - \frac{1}{4\pi\sqrt{-g}} \partial_\rho (\sqrt{-g} \tilde{A}_\tau \tilde{F}^{\rho\mu}) \tag{51}$$

$$\mathcal{J}^\mu = T^\mu{}_\varphi - \frac{1}{4\pi\sqrt{-g}} \partial_\rho (\sqrt{-g} (\tilde{A}_\varphi - Q_m C) \tilde{F}^{\rho\mu}). \tag{52}$$

The additional terms $\frac{-1}{4\pi\sqrt{-g}}\partial_\rho(\sqrt{-g}\tilde{A}_\tau\tilde{F}^{\rho\mu})$ and $\frac{-1}{4\pi\sqrt{-g}}\partial_\rho(\sqrt{-g}(\tilde{A}_\varphi - Q_m C)\tilde{F}^{\rho\mu})$ are of the form $\nabla_\nu f^{\mu\nu}$, where $f^{\mu\nu}$ is antisymmetric. Currents of this form are automatically conserved regardless of the value of $f^{\mu\nu}$. It is not difficult to verify using partial integration that these terms do not give any contribution to dM and dJ , therefore the definitions (51) and (52) also lead to the results (49) and (50).

We also note finally that the presence of the scalar field is not essential in the derivation above. If it is omitted, then the case of a purely electromagnetic test field is obtained.

Appendix. Noether's theorem

Let the action of a physical system described by a collection of real fields $\Phi_i(x^a)$ on an n -dimensional spacetime be

$$\mathcal{S} = \int dx^1 dx^2 \dots dx^n \mathbf{L}(\Phi_i(x^a), \partial_b \Phi_i(x^a), x^a), \quad (\text{A.1})$$

with Lagrangian $\mathbf{L}(\Phi_i(x^a), \partial_b \Phi_i(x^a), x^a)$. The equations of motions are the Euler-Lagrange equations

$$\frac{\partial \mathbf{L}}{\partial \Phi_i} - D_\mu \frac{\partial \mathbf{L}}{\partial (\partial_\mu \Phi_i)} = 0. \quad (\text{A.2})$$

The notation D_μ is used for the total derivative with respect to x^μ . If, for example, f is a function of x^a , then $D_\mu f = \partial_\mu f$, whereas for a function $f(\Phi_i, x^a)$ we have $D_\mu f = \frac{\partial f}{\partial \Phi_i} \partial_\mu \Phi_i + \frac{\partial f}{\partial x^\mu}$.

Assume that Φ_i satisfy the Euler-Lagrange equations, and the invariance condition

$$\frac{\partial \mathbf{L}}{\partial \Phi_i} \Delta \Phi_i + \frac{\partial \mathbf{L}}{\partial (\partial_\mu \Phi_i)} D_\mu (\Delta \Phi_i) = D_\mu K^\mu \quad (\text{A.3})$$

holds with some functions $\Delta \Phi_i$ and K^μ . $\Delta \Phi_i$ denote the change of the fields under an infinitesimal transformation $\Phi_i \rightarrow \Phi_i + \epsilon \Delta \Phi_i$. The expression on the left hand side is the change of \mathbf{L} under this transformation. Now it is straightforward to see, using (A.2) and (A.3), that the current

$$j^\mu = \frac{\partial \mathbf{L}}{\partial (\partial_\mu \Phi_i)} \Delta \Phi_i - K^\mu \quad (\text{A.4})$$

is conserved, i.e.

$$D_\mu j^\mu = 0. \quad (\text{A.5})$$

This theorem is independent of any metric structure on the spacetime manifold.

In section 4 we have $\mathbf{L} = \sqrt{-g}\mathcal{L}$; for time translations

$$\Delta \psi = -\partial_\tau \psi, \quad \Delta \psi^* = -\partial_\tau \psi^*, \quad \Delta \tilde{A}_\mu = -\partial_\tau \tilde{A}_\mu, \quad K^\mu = -\delta^\mu_\tau \sqrt{-g}\mathcal{L}; \quad (\text{A.6})$$

for rotations

$$\Delta\psi = -\partial_\varphi\psi, \quad \Delta\psi^* = -\partial_\varphi\psi^*, \quad \Delta\tilde{A}_\mu = -\partial_\varphi\tilde{A}_\mu, \quad K^\mu = -\delta^\mu_\varphi\sqrt{-g}\mathcal{L}. \quad (\text{A.7})$$

For global $U(1)$ gauge transformations we have

$$\Delta\psi = i\psi, \quad \Delta\psi^* = -i\psi^*, \quad K^\mu = 0. \quad (\text{A.8})$$

The invariance condition is satisfied for any fields in these cases, not only for the solutions of the Euler-Lagrange equations.

Acknowledgment

The author would like to thank István Rácz for useful discussions.

References

- [1] Penrose R. 1969 Gravitational collapse: the role of general relativity, *Riv. Nuovo Cimento* **1**, special number, 252
- [2] Wald R. M. 1984 *General relativity*, University of Chicago Press, Chicago
- [3] Wald R. M. 1974 Gedanken experiments to destroy a black hole *Annals of Physics* **83** 548
- [4] Needham T. 1980 Cosmic censorship and test particles *Phys. Rev. D* **22** 791
- [5] Hiscock W. A. 1981 Magnetic charge, black holes and cosmic censorship *Ann. Phys.* **131** 245
- [6] Semiz I. 1990 Dyon black holes do not violate cosmic censorship *Class. Quant. Grav.* **7** 353
- [7] Semiz I. 2011 Dyonic Kerr-Newman black holes, complex scalar field and cosmic censorship *Gen. Relativ. Gravit.* **43** 833
- [8] Wu T. T., Yang C. N. 1976 Dirac monopole without strings: monopole harmonics *Nucl. Phys. B* **107** 365
- [9] Mazur P. O. 1982 Proof of uniqueness of the Kerr-Newman black hole solution *J. Math. Phys.* **15** 3173
- [10] Bunting G. L. 1983 Proof of the uniqueness conjecture for black holes, Ph.D. Thesis, University of New England, Armidale, Australia
- [11] Wald R. M. 1993 Black hole entropy is the Noether charge *Phys. Rev. D* **48** R3427

- [12] Wald R. M. 1994 *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press, Chicago, section 6.2
- [13] Iyer V., Wald R. M. 1994 Some properties of the Noether charge and a proposal for dynamical black hole entropy *Phys. Rev. D* **50** 846
- [14] Gao S., Wald R. M. 2001 “Physical process version” of the first law and the generalized second law for charged and rotating black holes *Phys. Rev. D* **64** 084020
- [15] Gao S. 2003 First law of black hole mechanics in Einstein-Maxwell and Einstein-Yang-Mills theories *Phys. Rev. D* **68** 044016
- [16] Nielsen A. B. 2009 Black holes and black hole thermodynamics without event horizons *Gen. Relativ. Gravit.* **41** 1539
- [17] Cohen J. M., Gautreau R. 1979 Naked singularities, event horizons, and charged particles *Phys. Rev. D* **19** 2273
- [18] Ford L. H., Roman T. A. 1992 ‘Cosmic flashing’ in four dimensions *Phys. Rev. D* **46** 1328
- [19] Bekenstein J. D., Rosenzweig C. 1994 Stability of the black hole horizon and the Landau ghost *Phys. Rev. D* **50** 7239
- [20] Jensen B. 1995 Stability of black hole event horizons *Phys. Rev. D* **51** 5511
- [21] Hod S. 1999 Black-hole polarization and cosmic censorship *Phys. Rev. D* **60** 104031
- [22] Hod S., Piran T. 2000 Cosmic censorship: the role of Quantum Gravity *Gen. Relativ. Gravit.* **12** 2333
- [23] Hod S. 2008 Weak cosmic censorship: as strong as ever *Phys. Rev. Lett.* **100** 121101
- [24] Hod S. 2008 Return of the cosmic censor *Phys. Lett. B* **668** 346
- [25] Gao S., Lemos J. P. S. 2008 Collapsing and static thin massive charged dust shells in a Reissner-Nordström black hole background in higher dimensions *Int. J. Mod. Phys. A* **23** 2943
- [26] Bouhmadi-Lopez M., Cardoso V., Nerozzi A., Rocha J. V. 2010 Black holes die hard: can one spin up a black hole past extremality? *Phys. Rev. D* **81** 084051
- [27] Barausse E., Cardoso V., Khanna G. 2010 Test bodies and naked singularities: is the self-force the cosmic censor? *Phys. Rev. Lett.* **105** 261102
- [28] Barausse E., Cardoso V., Khanna G. 2011 Testing the Cosmic Censorship Conjecture with point particles: the effect of radiation reaction and the self-force *Phys. Rev. D* **84** 104006

- [29] Isoyama S., Sago N., Tanaka T. 2011 Cosmic censorship in overcharging a Reissner-Nordström black hole via charged particle absorption, *Phys. Rev. D* **84** 124024
- [30] Shapiro S. L., Teukolsky S. A. 1991 Formation of naked singularities: the violation of cosmic censorship *Phys. Rev. Lett.* **66** 994
- [31] Hubeny V. E. 1999 Overcharging a black hole and cosmic censorship *Phys. Rev. D* **59** 064013
- [32] de Felice F., Yunqiang Yu 2001 Turning a black hole into a naked singularity *Class. Quant. Grav.* **18** 1235
- [33] Matsas G. E. A., da Silva A. R. R. 2007 Overspinning a nearly extreme charged black hole via a quantum tunneling process *Phys. Rev. Lett.* **99** 181301
- [34] Jacobson T., Sotiriou T. P. 2009 Over-spinning a black hole with a test body *Phys. Rev. Lett.* **103** 141101; *Phys. Rev. Lett.* **103** 209903 (Erratum)
- [35] Matsas G. E. A., Richartz M., Saa A., da Silva A. R. R., Vanzella D. A. T. 2009 Can quantum mechanics fool the cosmic censor? *Phys. Rev. D* **79** 101502(R)
- [36] Richartz M., Saa A. 2011 Challenging the weak cosmic censorship conjecture with charged quantum particles *Phys. Rev. D* **84** 104021
- [37] Saa A., Santarelli R. 2011 Destroying a near-extremal Kerr-Newman black hole *Phys. Rev. D* **84** 027501
- [38] Virbhadra K. S., Ellis G. F. R. 2002 Gravitational lensing by naked singularities *Phys. Rev. D* **65** 103004
- [39] Virbhadra K. S., Keeton C. R. 2008 Time delay and magnification centroid due to gravitational lensing by black holes and naked singularities *Phys. Rev. D* **77** 124014
- [40] Wald R. M. 1997 Gravitational collapse and cosmic censorship, gr-qc/9710068
- [41] Joshi P. S. 2002 Cosmic censorship: a current perspective *Modern Phys. Lett. A* **17** 1067
- [42] Clarke C. J. S. 1994 A title of cosmic censorship *Class. Quant. Grav.* **11** 1375
- [43] Singh T. P. 1999 Gravitational collapse, black holes and naked singularities *J. Astrophys. Astron.* **20** 221, gr-qc/9805066
- [44] Krolak A. 1999 Nature of singularities in gravitational collapse *Prog. Theor. Phys. Suppl.* **136** 45